AN L. P. APPROACH TO SHORT RUN BUS SYSTEM CHANGES

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By

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ABSTRACT

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This research is concerned with short run changes to urban bus systems to help improve level of service and profit. The short run context of this problem limits the scope of feasible changes to the most basic and easily implemented. Specifically, a method is developed which uses the computational power of linear programming and the analytical ability of the systems analyst. Linear programming is used, not as a direct means to the optimal solution, but rather as a tool to provide a sensitivity analysis. The analyst, on the other hand, uses the information of this post-optimality analysis upon which to base his recommendations.

First, demand is addressed and a simplistic formulation predicting bus ridership as a function of frequency is developed. Next, the system formulation is presented, including all relevant facets of urban bus transit. From the interpretation of the computer reports of the linear programming solution to the formulation the most advantageous short run changes may be proposed.

The method provides a rapid and useful means of determining short run alterations. The data requirements are not excessive, but do require the careful consideration of any potential users in order to insure the accuracy of the model.

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CHAPTER I

INTRODUCTION TO BUS SYSTEM CHANGES

1.1 Introduction

This thesis examines the application of linear programming to alter an existing bus system. The bus network of a mass transit system, although in financial difficulties, is important to many urban travellers. (38) For many communities buses provide the only public transportation services, as well as being the only means of travel for the very old, young or handicapped. In order to help in maintaining these services, short run changes must be implemented, as demand patterns change, to improve the profitability of such systems. The determination of these changes is the topic of this thesis. Linear programming is used as a tool in evaluating an existing system and leading the decision maker in the direction of desirable changes. By the careful use of information developed by this method, temporary relief for some of the financial pressures may be realized.

1.2 Discussion of Bus Service

In the past, automobile ownership was a status symbol as well as a means of transportation. Before the compact car boom of the 60's this was evidenced by the automobile manufacturers propensity to build larger and more sophisticated cars. As auto sales increased, more high-

ways and expressways were built to accommodate the added volume until now when land in urban areas for additional roadways is scarce. The automobile has provided the flexibility, status, comfort and convenience for which it was designed, but today, it increasingly provides congestion, pollution and inconvenience due to the scale of its success.

Recently the automobile has fallen into disfavor as the primary means of urban transport. (31) The reasons for this are many and varied, however, they revolve around the peak hour congestion seen on todays highways leading to and from urban areas. The aggravation of sitting in a traffic jam twice a day has turned commuters against the automobile. As interest turns to mass transit, it becomes apparent that in most such networks the bus system most frequently operates at a loss. (24) The recent (1968-1969) decline of passenger volume on regular route bus service has been 1.2% while the change in labor cost for the same period has been an increase of 4.5%. (24) Also, as Owen states (26), "The downward trend in transit riding since the war has been so much more destructive to the industry than total traffic figures indicate. The mass transportation problem is magnified by the fact that traffic has continued high during peak hours of the day, while most of the loss of business has been during the off peak hours." This fact is dramatically illustrated by the 38% decline in transit riders during the period 1946 to 1953 while containing only a 10% peak hour decline. (26) Today, although the overall decline in ridership

has been stopped, the peaking phenomenon still exists and is causing acute financial problems. (1)

The bus service of a mass transit network, exhibits all the above phenomena but also has unique problems. The nature of a bus system, in itself, adds to the monetary problems. A common characteristic of the bus service in most cities has been a downward spiral in rider volume, accompanied by fare increases and decreasing levels of service. This in turn, further decreases volume and continues the spiral until only inadequate service exists and it is used only by those who have no effective alternatives. (30)

Unlike rail systems, additional seats are not easily available (in the form of extra cars) to cope with peak demand at low marginal cost; buses require an added driver and the addition of all associated operating costs for each extra bus in service. "To provide extra bus services, at the peak along a particular route may require the provision of extra buses, garage accommodations, drivers, and all other requirements of providing additional capacity. Assuming these extra costs must be covered by revenues earned, the latter must be sufficient to cover not only those variable costs of operation which depend on vehicle-miles worked, such as petrol, oil and some wear and tear, but also such time or capacity costs as interest and depreciation on the capital expenditure involved, rent and rates on extra garaging accommodation, vehicle licenses and insurance, and the full costs of employing additional drivers. The crux of the matter being that in so

far as the demand for the services to be provided is confined to a very brief duration, virtually all the revenue required to meet both time and variable costs may have to be earned during that short time."(27)

What this all indicates is that public transit operators with fixed facilities require a physical plant expansive enough to meet peak demand. Beyond this, the public transit operator must invest in sufficient equipment to handle peak demand, plus extra facilities to store and maintain that equipment. It is not surprising that much of the financial difficulty of public transport operators arises from the fact that so much of the plant and equipment must stand idle so much of the time. A new air-conditioned bus, depending on its size, costs approximately $25,000 to $35,000. When such expensive hardware is used only about four hours a day, it is very difficult for a privately owned firm to justify the investment or, for that matter operate at a profit. (33)

Labor is another expensive problem. For example, in a dial-a-ride system the labor costs account for approximately 47% of the total costs. Standard union contracts usually call for a full day's pay whenever an employee is called to duty. In off-peak hours it is often almost impossible to fully utilize the manpower needed to handle rush hour traffic. As a result, men are frequently paid for hours they do not work. (33)

This inability to utilize facilities, equipment and manpower

more fully is further aggravated by the decline of off peak traffic. Not only has the daytime traffic declined, but evening and weekend travel to the urban core for entertainment or recreation has fallen off markedly in the last twenty years. Television tends to keep people at home. The pattern of increased suburban living has acted to concentrate off-peak travel in suburban locations, encouraging a type of traffic that is best suited to the private automobile. (33)

Further, unlike the air traveller, the transit rider seldom arrives at a stop with prior knowledge of bus arrival time. Transit riders do not attempt to meet a bus schedule, but rather arrive at a stop hoping that the wait won't be too long. If headways are too large and the rider expects a shorter wait time, he will find another mode more suitable to his needs.

Clearly, the urban crisis has been recognized by the federal government, as indicated by speeches by late President Kennedy (37) and Senator Williams. (35) Beyond recognition, the government has acted (as early as 1964) to alleviate the problems of urban transportation. (40, 2) The majority of these actions involve long run studies, alterations and improvements. This provides a solution to the problem in the future but action must be taken today to maintain bus service and mass transit in general as a viable alternative to the automobile.

This is the problem; short run changes to bus schedules and systems are essential if adequate economically viable service is to be maintained. (34) The determination of these changes as demand shifts

occur and passenger preferences change is the initial problem. The demand shifts are caused by many factors such as changes in the socio-economic characteristics of an area, reliability of newer automobiles and level of service of the bus system. Passenger preferences are behavioral in nature and very difficult to model and predict. Management, in the face of the industry's existing problems, has been reluctant to accept new technology, revamp the historic route network (which in many cases follows old street car routes), or in general make necessary or desirable changes. Public opinion also plays an important role in this problem. The complete reorganization of a bus system, although increasing level of service and profit, may in fact be so disconcerting to users, that public outcry stops any such proposed action. This, plus union and company policies (such as the "NO HIRE - NO FIRE" policy) force any system changes to be incremental.

Short run changes provide an immediate solution to the problem but may not provide any assistance in the long run context of today's bus problem. This thesis approaches the problem only from the short run point of view. It provides a useful method for the establishment of these immediate changes in an attempt to reduce the financial problems. This temporary step is important in order to "buy time" to help prepare for long run changes.

1.3 Alternative Methods of Analysis

At present three methods are available to help determine where these network changes should be made:

1. Full scale analysis
2. Computer simulation
3. Linear Programming

The first, and most costly, is the full scale study of the system in question. This type of analysis is the most thorough in that it considers the network, region and all relevant factors. Lampkin and Saalmans have completed this type of in depth analysis in an attempt to reorganize the route structure, establish new frequencies along routes and provide precise timetables in order to run a financially feasible bus service with a minimum of active buses in a small North England town. (21) It is an in depth look at all factors in an attempt to specify an optimal solution. Beyond this, Boston is on the verge of a major transport study which will be very useful. Other full scale studies have been made concerning new technology (36) and the establishment of new types of service. (15) From the results of such a study of a bus system, recommendations may be made based on the relevant measures of effectiveness. In short, an in depth analysis attempts to look at all factors in order to bring out all impacts that a variety of alternatives will cause.

This type of analysis provides a great deal of useful information and is most helpful to the decision-maker. However, it has major drawbacks when being used in a short run analysis. The cost is very high, in most cases prohibitive, for the short run and the study itself requires quite a long time to complete. If changes are desired next month, management cannot, and will not wait six months to a year to obtain a thorough study. Also, the information generated in such a study is useful but will often have much broader applications. It is almost wasted if used only for this type of problem. In effect a thumb tack is being driven with a sledge hammer.

A second approach is through computer simulation. Simulation involves the use of a computer to simulate the individual actions of the network in order to give management a more direct grasp of the problem areas. Once a simulation is developed it provides a superficial view of the results of subtle system interactions. Through this type of analysis, feasible alternatives are developed and tested by rerunning the simulation. The resultant system can now be re-evaluated from a "results produced" aspect and a determination of the effectiveness of the solution used may be made.

Simulation methods have been used in many diverse fields ranging from garbage collection (39) to passenger and aircraft flows in an airport. (9) The most recent, and also closest application to the problem at hand, has been by Kulash (20) who uses simulation to analyze fixed route public transport systems. These methods provide a

simple handle on the entire system and its workings. Using this technique, alternatives may be tested at relatively little cost or capital expenditure while the results are valid predictions of what will happen. Briefly, a manager can test any number of alternatives or simply experiment with various ideas at a low cost in hopes of striking upon a suitable solution or improvement.

Questions arise in the validity of such models. Do they accurately reflect what is happening? The usefulness of a simulation hinges entirely on the assumptions made by the modeller. For example, is a random distribution of passenger arrival times valid? Does a passenger actually behave as the modeller has assumed? The sensitivity of simulation to these factors tends to reflect the biases and interpretation of the modeller. In this way model biases may slant a model towards a solution with the decision-maker unaware of it. Further, this type of model only demonstrates the resultant actions of a system, it provides no insight into the subtle but relevant inter-relationships which govern these actions. It gives no real guidelines, save showing areas of congestion or low revenue, that lead the way to the most effective solution. When employing a simulation the method is merely a sophisticated trial and error single situation analysis technique which hopefully leads to a good solution.

A third method, linear programming has become a widely used alternative computer method for solving problems of this nature.

"Problems which seek to maximize or minimize a numerical function of a number of variables (or functions), with the variables subject to certain constraints, form a general class which may be called optimization problems. Linear programming deal with that class of optimization problem for which all relations among the variables are linear. The relations must be linear both in the constraints and in the function to be optimized." (16)

Linear programming formulations have been used in many diverse areas in the transportation field such as, establishing a rotating crew roster for a bus system, (3) spacing transit stations for minimum travel time or maximum passengers, (42, 43) establishing coordinated bus schedules for routes which have a common link, (11) routing of utility vehicles which operate from fixed depots (13), and minimizing the cost of refuse collection operations. (6) Beyond this, formulations have been extended to include dynamic programming algorithms for the development of schedules for passenger transport service. (44)

As the topic of this paper, linear programming provides a useful and very informative tool in approaching the short run bus system problem of incremental changes. Linear programming is not actually a computer procedure but rather a mathematical optimization technique that lends itself quite readily to computer solution. The computer enables large problems to be quickly solved and highly relevant information developed.

The basic concepts of linear programming give rise to the dual variables and shadow prices, which are exceptionally good indicators of sensitivity. Specifically, the method of this thesis provides a sensitivity analysis of the system modelled and shows the relationships between relevant factors. The objective may be profit maximization or cost minimization and the results provide the changes in profit or cost associated with unit changes in the various constraining factors. This gives the decision-maker not only the necessary guidelines as to where change is required, but also the quantitative amounts associated with all possible changes.

The validity of this method depends primarily on the system demand and problem formulation. Further, results must be properly interpreted in order to produce workable solutions. From these results feasible and effective alternatives may be developed and implemented.

1.4 Summary

In summary, the problem of determining the proper changes to a bus system may be approached historically in three ways. Through a full scale study of the system, which is costly and time consuming, a computer simulation which only amounts to a hit and miss method, and linear programming. Linear programming provides an efficient means of obtaining a sensitivity analysis relating the system actions to the various factors. It points the way for change and gives quantitative

amounts that can be expected if certain changes are implemented. Problems develop in the demand modelling (Chapter II) and actual formulation (Chapter III) but they can be solved (or side-stepped) in order to allow the use of this method. The interpretation of results is the crux of the method and is presented in Chapter IV. Finally, summaries and conclusions are found in the last chapter.

This thesis is a first step in this application of linear programming. The capabilities and powers of L.P. have been used in various fields, and increasingly today it is helping in the solution to the urban transportation problem. By using it in the short run context, immediate results are possible in order to relieve some of the financial pressures. It must be stressed, however, that this method is not the ultimate solution, rather, it merely provides a useful tool to help management and the decision-maker make beneficial and profitable decisions.

CHAPTER II

DEMAND FOR BUS SERVICE

2.1 Introduction

To analyse any transport system a basic understanding of the demand for that system is necessary. The demand is composed of individuals, each having his own set of values and desires, so in order to fully describe a system which depends on each rider, the aggregate of these sets of values must be discussed. This aggregate demand for the system requires a thorough understanding of the underlying factors which affect the riders, to provide the insight necessary to fully understand choice.

This chapter provides a simple formulation of the short run demand for urban bus services. It prunes burdensome variables leaving an efficient and effective model for estimating bus demand.

2.2 Discussion of Bus Demand

Dictionaries define demand as a requirement or need for a commodity or service. Undoubtedly, the demand for transportation in the urban sector is a real and important shaping force for the community. In any analysis concerning the alteration of the transport system it must be carefully considered. However, the factors that govern this demand are neither clear cut nor obvious. Trips are parts of larger activities, such as working, shopping or vacationing, they are

seldom ends in themselves and therefore involve many factors that are not readily apparent. For these reasons demand for transportation is classified as a derived demand.

Oi and Shuldiner (25) describe a derived demand using film as an illustrative example. A purchaser of film usually requires not only the package of film but also its development after exposure. In this way the two components are jointly demanded, the demand for film (pictures) deriving a demand for processing. Other examples are automobiles and gasoline, labor and capital, and freight services and commodities shipped. In short, the essential feature is the high level of complementarity between two jointly demanded components.

This concept is clearly applicable to the individual components of a transportation network. Of particular interest, private automobiles and transit exhibit demand of this nature. Oi and Shuldiner (25) go on to state that transit trip generation is highly dependent on many factors such as household size, availability of alternative modes, distance from the Central Business District (CBD), and car ownership. Looking first at household size it is clear that as a family enlarges in size more trips are generated. Children must go to and from school or work, and more shopping trips occur. Since the children usually cannot drive and don't own their own cars they will most likely ride transit.

Since the trip generation and modal choice process are one and the same, the availability of alternative modes plays a great role in

this process. If many alternative modes are available, the traveller can choose the mode most suitable to his needs. This choice will result in ridership volumes on modes closely related to the levels of service offered by each mode. The alternative modes may differ merely in the feasibility that the user perceives. A man with a car living close to the CBD may not feel the automobile is a feasible or attractive mode to choose. The traffic congestion or trip length to the CBD may discourage some trips from being made, if they may be postponed or eliminated. Trips may be put off or eliminated by going to another destination to satisfy the trip purpose. This also shows that the distance from the CBD plays an important part in transit trip generation and modal choice since transit availability is reduced as you go further from the CBD.

Car ownership itself is the major factor in transit trip generation according to Oi and Shuldiner. (25) If a family owns a car this immediately is an alternative mode and depending on how many cars are available, changes the tripmaking patterns. A home with one car which the husband uses for commuting to work, will produce daily shopping and school trips by other modes, primarily transit. On the other hand, if the breadwinner uses transit for his work trip, the car is available for those other trip purposes. Similarly, if a second car is available it further reduces the propensity to use transit. However, if no car is available then all trips must be geared to a "riding" mode. A riding mode being one in which the user is not

responsible for the operation of the vehicle. For example, transit or car pools.

Oi and Shuldiner (25) extend this concept to define a term called "base demand for transit" as a demand for transit irrespective of the number of autos at the family's disposal. This is to say that some trips generated by a family will be transit trips even if other modes are available. This term identifies the trip makers who will ride transit as long as its service exists. This indicates that such a rider will alter his schedule in order to meet transit schedules and may even go out of his way to reach a transit stop.

Hoel et al (18) have described "latent demand for transit" as the potential trips that would arise if a means of transportation or higher level of service was available. The groups who actually make up the latent demand and who are totally dependent upon others for transportation consist of:

a) The elderly who cannot or choose not to drive

b) The young

c) The secondary worker

d) Housewives

e) Poor persons

f) The handicapped.

With these two concepts, facts from studies, actual experience and an intuitive feel for transit operations, demand functions may be proposed. Ideally, it would be useful to have an accurate and

detailed estimate of the bus transit demand function, including all price and income elasticities and cross elasticities. Unfortunately, very few studies have been completed in this area and the data is very scarce. Although there have been some useful studies, they are too broad to be applicable to the limited scope of this problem. A prime example in this area is the CATS study. (5)

Looking specifically at bus transit one sees that its major use is for work, shopping and school trips, while recreation trips are seldom bus oriented. (5) Now that the car is a part of nearly every household, recreation lends itself most readily to the auto mode rather than public transit. This is because the car provides comfort and flexibility. A family need not preplan recreation since the car can be driven nearly anywhere and does not impose any fixed schedule. Public transit, on the other hand, follows fixed routes and schedules and therefore hinders even the simplest trips with transfers, waiting time, limited luggage space and inconvenience.

This restriction of trip purpose on buses is what causes the peaked nature of bus transit. The peak hours always see crowded buses while during off peak hours buses are often practically empty. This fact is evidenced by Lassow, (22) who divided the day into five periods and determined the percent transit usage during each period. The data is for New York City transit, which may be atypical, but clearly shows the trend.

PERIODS	PERCENT DISTRIBUTION
0700 - 1000	31
1000 - 1600	21
1600 - 1900	30
1900 - 2300	10
2300 - 0700	8
	100

TABLE 2.1 Transit Usage By Time Period

The peaking phenomon shown above indicates that the time period over which an analysis is made is important to modelling accuracy. These variations in demand must be modelled in the demand formulation.

2.3 Formulation of a Demand Function for Bus Service

Modal split studies (10) have indicated that transit ridership, particularly bus usage, is highly dependent on the level of service provided. Kraft and Domencich (19) state that transit ridership is responsive to improvements in service, or:

Bus Ridership = F (Level of Service).

More clearly, level of service is composed of many components; travel time, fare, frequency of service, waiting time, transfer time, walking distance and comfort and convenience. All of these factors play an important part in determining what any one person perceives

as a level of service. Also, these factors are all interrelated in describing levels of service for a group as well as individuals.

First, looking at travel time it is apparent that a certain group of riders (those who value time greatly) will be more sensitive to time considerations than, say, fare changes. Conversely, others who have different values may feel that travel time is not a primary consideration. This argument may also be extended to transfer time and walking distance. Again, riders with high values of time tend to choose modes that provide a high level of service, or short travel and transfer times. Similarly, a rider with a handicap or someone with difficulty walking would be most sensitive to walking distance.

These factors cannot be altered in the short run. They may be improved through the complete reconstruction of the route network, however, managerial and public opinion and regulation usually preclude this. Also, these types of changes are extremely costly and not feasible in the short run context. For these reasons they will be considered fixed. It must also be noted, however, that through other changes, such as increases in frequency of service, overall transfer and travel time are reduced through the reduction of wait times. These effects are of secondary nature and will not be considered here.

Many riders are highly aware of the lack of comfort or convenience of bus transit. For example, the three steps into a bus can

become a hindrance to the aged or handicapped, also air conditioning has been praised in the more modern buses. During the summer months the rush hour bus commuter sees this as a godsend. The convenience of not "fighting traffic" along with comfortable seats and a cool ride may be factors that influence many in modal choice. Clearly, the ability to improve comfort depends on bus technology and the financial ability to obtain new equipment. New buses are not easily obtained items at a cost of over $30,000. The acquisition of new buses on the short run basis is impossible and for this reason the available buses are to be assumed fixed in terms of number, comfort and convenience.

When attempting to analyse a system for short run changes the elasticities are critical. Elasticity is defined as a relationship between a dependent variable such as ridership or profit and one of the component independent variables like fare. More precisely, the elasticity represents the percent change in the dependent variable when a one percent change in the independent variable occurs. This is vital to the analysis because if, for example, an increase in fare will reduce ridership to the extent that revenue is lost, then it should not be increased. Similarly for level of service, if a reduction in service will reduce costs but disproportionally reduce ridership, again the change should not be made.

Looking specifically at fare, superficially it would appear that it is a major factor in transit ridership, however, analysis has

not shown this to be true in the past. Kraft and Domencich (19) have estimated the elasticities for transit riderships with respect to fares as:

ELASTICITIES WITH RESPECT TO COST (FARE)

TRIP PURPOSE	TRANSIT LINE HAUL	TRANSIT ACCESS
Work	-. 09	-. 10
Shopping	-. 323*	

*Available shopping transit trip sample was unsuitable for estimating elasticities for the disaggregated cost components.

TABLE 2.2 TRANSIT RIDERSHIP ELASTICITIES WITH RESPECT TO FARE

Curtin (7) has summarized transit fare changes for the past 20 years and has drawn up Table 2.3. The factor "loss ratio" is defined as the rate of passenger loss attributable to fare increases. It is applied to the present fare increases to determine the rate of passenger loss as a result of the higher price. The columns percent fare increase and percent ridership loss have been added by the author in order to more clearly indicate the phenomenon.

FARE INCREASES

DATE	CITY	FROM	TO	% FARE INCREASE	LOSS RATIO	% RIDERSHIP LOSS
June 1952	San Francisco	10	15	50	.18	9.18
July 1953	New York City	10	15	50	.20	10
July 1966	New York City	15	20	33 1/3	.18	6
Oct 1955	Boston (Surface)	13	15	6 1/2	.19	
	(Rapid)	18-20	20	9		
Feb 1958	Portland	20	25	25	.28	7
Dec 1963	Salt Lake City	20	25	25	.12	3
June 1958	Connecticut Co.	15	20	33 1/3	.28	9.3
Oct 1963	Atlanta	20	25	25	.28	7
July 1957	Cincinnati	20	25	25	.24	6
Jan 1954	Philadelphia	15	18	20	.14	2.8
Oct 1958	Baltimore	20	25	25	.08	2
Jan 1954	New York City (Bus)	10	13	13 1/3	.30	10
July 1957	Chicago	20	25	25	.30	7.5

TABLE 2.3 TRANSIT FARE CHANGES

Beyond this, Carstens and Csanyi, (4) in their transit usage model for Iowa cities state that the price elasticity of revenue (which is a surrogate for ridership) depends on the level of service. At low service levels, transit ridership is more sensitive to fare changes than at high service levels. In nearly all of today's urban

transit systems the level of service is high enough to cause ridership to be relatively inelastic with respect to price.

In New York City (22) the 1966 fare increase of 33 1/3 percent produced an average ten percent decrease in ridership on bus transit. This figure is slightly higher than that presented in the earlier table because the table's figures included rail transit.

Therefore, all evidence points towards the fact that ridership is relatively insensitive to fare changes. Another important consideration which must be presented here is that in most cities a fare change is by no means a feasible short run alteration. Fare increases and decreases must be approved by various advisory and regulatory groups and in some cases by the city governments. In this way, fare changes, which appear simple and easy to invoke, may actually take quite long and be very difficult to implement. For this reason fare may be considered fixed in the short run for most areas, depending on the procedure for obtaining fare changes.

Considering these facts, fare may also be eliminated from the factors affecting demand in the short run. Finally, since waiting time is actually a function of frequency of service this leaves only frequency:

$$\text{Bus Ridership} = f\ (\text{Frequency})$$

Kraft (19) has implied this when he states that transit ridership is more responsive to improvements in service than to reduction in fare.

Proceeding from this in considering a single route, it is evident that as frequency of service increases along the route so also will ridership. Common sense dictates, however, that even if continuous service was available, hourly bus ridership would not increase monotonically, it would taper off and approach a limit. The additional riders are accounted for by latent demand as discussed earlier and by changes in modal choice. The notion of base demand, also discussed earlier, gives rise to the concept that as frequencies are reduced this also approaches a limit, the base demand. Again here, if frequencies are further reduced it is obvious that some of the riders will be so dissatisfied that they will not ride the bus along that route. This indicates that the base demand would decay to zero as the frequencies approached zero buses per hour. Graphically this becomes:

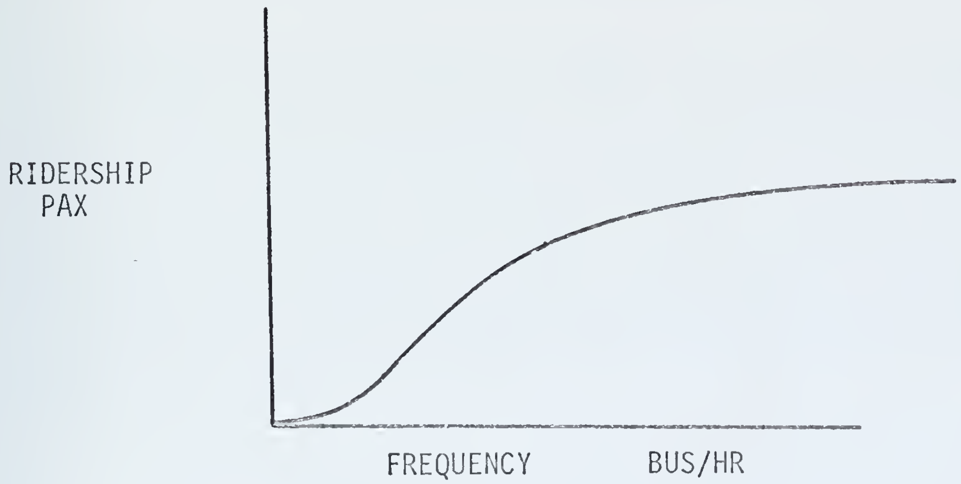

FIGURE 2.1 BUS ROUTE DEMAND

Since this analysis is geared to an "average" bus system, and since most systems operate well above this point of base demand decay, the lower portion of the curve will be neglected. Therefore, graphically the demand function along a single route will be:

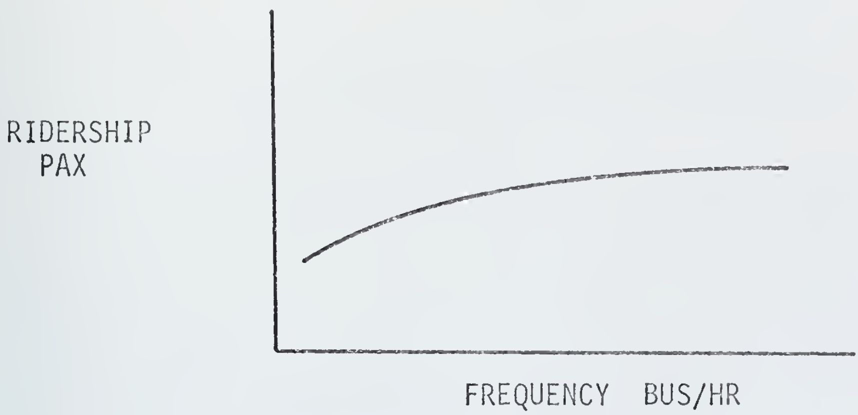

FIGURE 2.2 BUS ROUTE DEMAND (REDUCED)

Since each route is being considered individually for this analysis, any interactions that may exist between routes must be discussed. For example, if two routes serve the same or nearby destinations and originate relatively close together, the reduction or elimination of service along one route could cause an increase in ridership on the other.

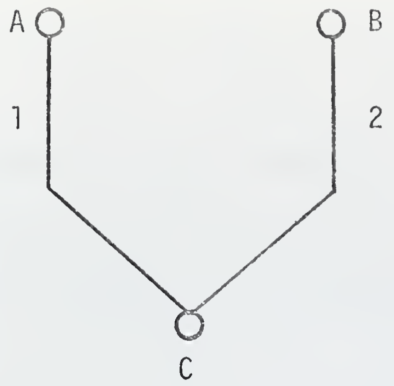

FIGURE 2.3 ROUTE INDEPENDENCE

If route two was cut or reduced in service, some of the riders might find it acceptable to walk from station B to A in order to ride along route one to complete their trip, thereby increasing the ridership along route one. This analysis assumes that no route will be completely cut and that all stations are sufficiently separated to insure the route independence of demand. In this way route interaction is eliminated, allowing this formulation to be accurate. The actual diversion of riders, even when service is not completely cut, and the effect of transfers is minimal when transfers are assumed as separate rides and the above assumptions are made. The validity of the formulation is not hindered in any way yet simplicity is maintained.

The independence of each route must also be considered when determining any unique features of a specific route. Travel characteristics and riders preferences will vary from route to route.

All of the diverse factors merely change the parameters of the curve in the above graph. For example, if along a route where minimal service exists the patential or latent demand is great the curve may look as follows (compared to a general curve passing through the same point at current frequency, shown dotted):

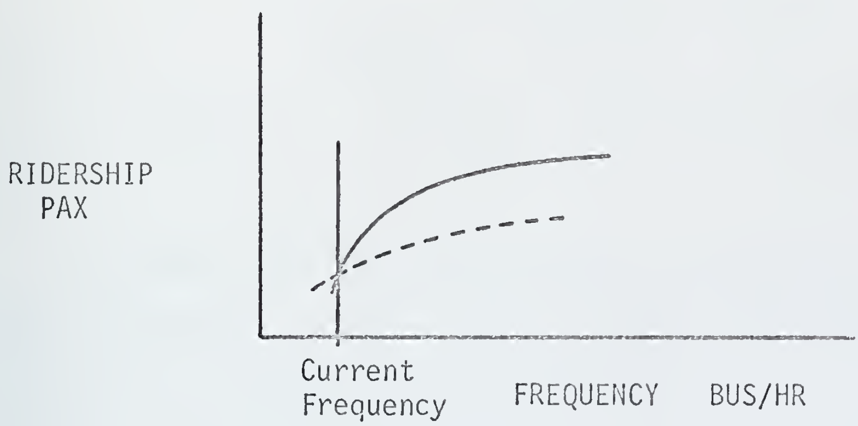

FIGURE 2.4 BUS DEMAND, MINIMAL SERVICE ROUTE

Similarly, if an area has very little demand for bus service, such as an outlying suburb, or during the early morning hours, the curve will be lower than the sample (dotted):

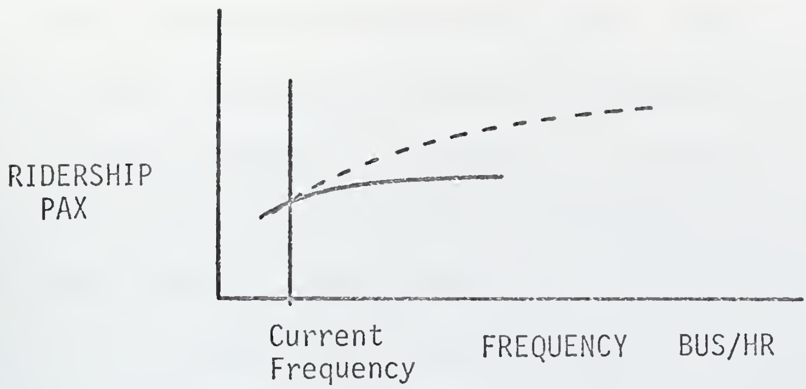

FIGURE 2.5 BUS DEMAND, MINIMAL DEMAND ROUTE

The above discussion points towards an exponential function which approaches a limiting value of demand. The function which most closely fits the description is as follows:

$$Ri = R_i^o + D_i \left[1 - e^{-ki(Xi - Xi^o)} \right] \qquad \forall i$$

where:

Ri = Ridership along route i (PAX)

Di = Theoretical Demand along route i if continuous service was available. The demand limit, i.e. headways of o or infinite frequencies (PAX).

Xi = Frequency of service along route i (BUS/HR)

Ki = Empirical constant which identifies each route i uniquely

e = Base of the natural logs

Xi^o= Existing frequency (BUS/HR)

Ri^o= Existing Ridership (PAX)

The K_i term needs further explanation. Above, the unique features of the various routes were discussed and shown to manifest themselves as a skewing or changing of the range of the demand curve. The K_i term is the factor that determines the skewing in order to model the various routes. Beyond this, the D_i term is the factor which determines the range of each demand function. In this way, through two empirical constants K_i and D_i each route, with its unique demand and ridership characteristics, is modelled accurately.

At first glance this function appears sufficient, however, due to the highly peaked nature of bus transit, it falls far short of its objective. It must be further subdivided into time periods to provide the necessary descriptive power required for this analysis.

Each day must be divided into m time periods, m depending on the individual characteristics of the system. This may be accomplished by subscripting the variables in the formulation again to represent each time period.

$$R_{ij} = \overset{o}{R}_{ij} + D_{ij} \left[1-e^{K_{ij}(X_{ij}-\overset{o}{X}_{ij})} \right] \qquad \forall ij$$

where:

R_{ij} = Ridership along route i during period j (PAX)

D_{ij} = Limiting demand along route i during period j (PAX)

X_{ij} = Frequency along route i during period j (BUS/HR)

K_{ij} = Empirical constant describing route i during period j

e = Base of the natural logs

Rij^{o} = Existing ridership along route i during period j (PAX)

Xij^{o} = Existing frequency along route i during period j (BUS/HR)

Kraft, Domencich and Valette (8) establish two time periods in their demand model, peak and off peak. A more realistic approach, since the demand is more time oriented (as shown in Table 2.1) is to use more time periods. The use of three periods will coincide most closely with travel patterns. Normal days see work trips during the peak hours, shopping trips between the peaks and miscellaneous work, shopping and recreation trips during the night hours. Therefore, establishing a peak period which occurs during the morning and afternoon rush hours describes the commuters rush to and from work very well. These riders have unique characteristics and must be treated separately from housewife shoppers or others. A second period, the day period, occurs between the two daily peak hours and is established to describe school, shopping and daytime recreation trips. Again, since this period's riders' desires, purposes and characteristics differ from other period's riders', a separate time period in the analysis is necessary. Finally a night period is also used, it is actually a catch-all for all late night and early morning bus riders. This period will always have lower ridership but is included for the sake of completeness. All night workers are accounted for here, as well as the various recreation trips such as to theaters and movies.

By describing three periods the model provides the precision necessary to give the analysis accurate results as well as a more concise breakdown of demands and areas that must be changed. However, certain costs are associated with the use of numerous time periods. For example, more data is required to accurately define each period and this means additional acquisition costs. Beyond this, the additional data and variable requirements adds to computer space and run time all increasing cost.

The formulation presented in this chapter is a simplified demand model in which additional sophistication would be excessive and unnecessary for the purpose at hand. This, when combined with the system formulation of the following chapter, provides the required accuracy so that the model may be used for analysis.

CHAPTER III

MATHEMATICAL MODEL OF A BUS SYSTEM

3.1 Introduction

The use of mathematical models to represent complex systems is increasing as more diverse systems are being modelled. In particular, linear programming because of its characteristics, is receiving increasing attention. From dual mode transportation systems, to product-mix problems, it is an accepted and well documented means of solving complex and difficult problems. This chapter develops a simple yet accurate model of an urban bus system which is suitable for a linear programming solution.

The system, consisting of fixed routes and time-tables, is modelled with the emphasis on profit and level of service. In this way, a clear understanding of system and factor interactions is developed. The formulation, although general in nature, is detailed enough to provide the necessary accuracy for proper interpretation. The scarcity of detail in certain areas is intended to allow the individual users of the model to add or change factors to mold the model to fit their own needs.

The formulation is actually a linear program in that all equations are linear and an optimal solution may be found. The optimal solution, however, is not that in the traditional sense, rather it is a representation of the existing system. In short, the

formulation contains certain constraints that limit the solution to be a mathematical representation of the current system, prior to any changes. The model itself represents the workings of the system and the solution is simply another such representation. The model in this chapter is not in any way intended to be a direct solution to the bus problem, but rather to produce a sensitivity analysis which only aids in the solution. At first glance, it may appear strange, to use such a powerful analytic tool to merely establish known facts, but its usefullness comes later in the interpretation of the associated outputs. The interpretation of these results, the dual variables and shadow prices, becomes the basis for analysis of the system. Through this interpretation, the sensitivity of the system to various factor changes is clearly shown.

The use of linear mathematical equations sometimes becomes difficult, since many phenomenon do not follow linear relationships. However, methods are available to obtain linear approximations from non-linear equations. Piecewise, linear approximation is one such method that allows linearization without too greatly sacrificing accuracy. This method will be used in the last portion of this chapter to linearize the demand function presented in chapter 2. It increases the number of variables but is necessary to allow the linear programing methods to be used.

3.2 Discussion of a Bus System

The bus system modelled is similar to nearly any existing urban bus transit network. Fixed routes exist and given frequencies of service are maintained along those routes. The routes themselves differ from each other only in trip time (the time required for a bus to complete a round trip along a route) and demand characteristics. The demand characteristics have already been discussed in chapter 2 and the differing trip time is simply an exogenous variable.

Daily time periods are established so that the peaking and congestion effects of traffic upon the system may be accurately represented. The use of these time periods allows the system to be completely described through trip time, ridership and frequency for any time during the day. The text of the preceeding chapter implies the use of three time periods, however, for generality, the number is left to the discretion of the user in the formulation of this chapter.

The buses operating in the system are assumed to be of a single type and capacity. Although buses have fixed seating capacities, during peak loading conditions these capacities may be far exceeded. For this reason maximum possible capacity is to be used as the capacity of each bus. Therefore, for the remainder of this chapter, the definition of capacity is the maximum number of riders that can be squeezed aboard a bus.

Buses are dispatched along routes with specific frequencies during

each time period. The responsibility for maintaining the frequencies at the dispatch point lies with the dispatcher. Along the route, good headway control maintains the time and spatial separation and keeps hourly frequencies constant. This headway control must be accomplished by the individual bus drivers unless starters exist at stops along each route. Since very few bus systems have this many starters the burden falls with the drivers. The bus driver has the timetable and may vary bus speed during off peak hours in order to meet the times indicated, however, when traffic flow is restricted and each stop is longer due to the increased number of passengers embarking and disembarking, the maintenance of proper headways becomes quite difficult.

Today this problem of poor headway control exists in nearly all bus transit systems during most time periods. Drivers tend to bunch up and do not maintain the established timetables. This fact affects the perceived frequency and has a detrimental affect on ridership. Methods exist to improve headway control but will not be presented here. This thesis is not addressing the question of headway control and therefore will neglect it. By assuming that good headways are maintained and that drivers adhere strictly to the timetables, the problem is side-stepped. The assumption is not correct, however, for the purposes of this analysis it is most useful.

Finally, since an unlimited number of buses are not available, certain constraints must be imposed to limit bus usage. The number of

buses available per hour may differ considerably from the total number owned since many will be unusable due to preventative maintenance, repairs or emergency service.

As brought out before, the system is general and allows many factors to be input by the user. It is apparent that with minor modifications the model will be applicable to nearly any urban bus system to allow diverse areas and interests to use it.

3.3 The Model of a Bus System

3.3.1 Capacity Constraints

In today's transit systems although older and less efficient equipment is being replaced by newer models, the average seating capacity of a bus has remained relatively constant. This seating capacity may be fixed at the number of seats, but due to standing and crowding, this number may be unrelated to the actual number of passengers that can be carried. For example, during the peak hours the actual capacity of a 42 seat bus may be well over 60 due to the crowded conditions. For this reason, the use of seating capacity as bus capacity would severly limit modelling actual operations of bus transit in this respect. Since each bus is capable of carrying a maximum load, the model bus capacity will be the maximum capacity.

From this discussion the assumption that each bus has a fixed capacity is made. If each bus has CAP places available and runs along a route i (during time period j) at frequency Xij Buses/Hr., clearly,

$$CAP \cdot Xij$$

places are available per hour along any route i during time period j.

The point along each route at which a bus is carrying the most riders is called the maximum load point. Clearly, the number of riders carried at this point cannot exceed the capacity of the vehicle. It is assumed that the route capacity (capacity per hour along a route) is sufficient to handle the volume at the maximum load point.

Now, separating those people who actually do ride the bus from those who would, one must differentiate between ridership and demand. If the demand ever exceeded the capacity, i.e., the volume at the maximum load point exceeded the route capacity, then ridership would be limited to be the capacity of the bus. However, the assumption above made this situation impossible to occur. It set the capacity

curve to the left of the demand function. This is clearly seen in the following graph:

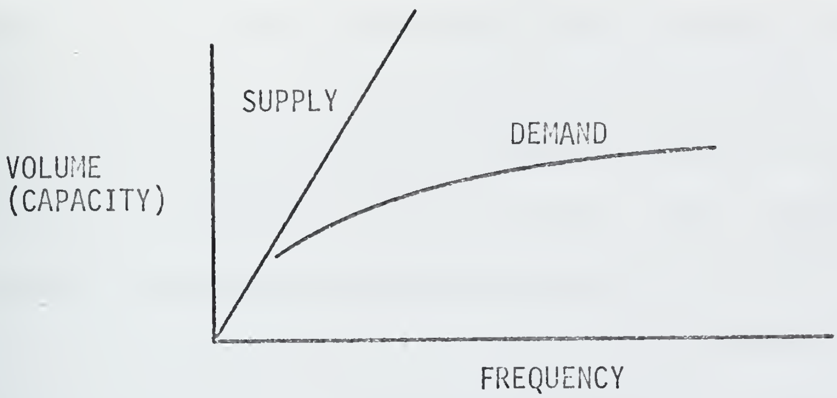

FIGURE 3.1 BUS SYSTEM SUPPLY AND DEMAND

Since the demand function never exceeds the capacity of a route, the ridership Rij (along route i during period j) will always be exceeded by the capacity. Simply, this means that on an hourly basis, all riders who arrive at a stop will be picked up by a bus during the hour. Mathematically, this becomes:

$$\text{CAP } X_{ij} \geq R_{ij} \qquad \forall ij$$

3.3.2 Bus Availability Constraints

In order to maintain the service that presently exists along all routes, a certain number of buses are used during each time period. Clearly this number never exceeds the number of buses

that are available. The buses available in turn, are determined by the schedules of maintenance, simple or major repairs and possible down time (time equipment is out of use due to a lack of parts or manpower) due to accidents or other mishaps. The availability of buses will vary among time periods since the demand differs between time periods. For example, since night demand is very low, many buses will be in the shops for scheduled maintenance and repairs and therefore, unavailable for use at night. Later, however, during the day when the demand increases these buses will be made available for use. This varying of bus availability between time periods is what leads to the use of a subscripted constant, Bj, which indicates the exact number of buses available during the jth time period.

Turning now to the number of buses in use, it is clear that enough buses are always available in order to maintain the schedules. Since the frequency Xij varies with each route and time period it is important to determine exactly how many buses are in use along each route. This, in turn, is dependent on the Round Trip travel time, Tij, along each route. Simply, for any route i (and time period j) the number of buses in use to maintain frequency Xij is:

$$Tij \cdot Xij$$

For example, if route i requires twenty minutes (1/3 hour) to complete during time period j, and has a frequency of six buses per

hour, then two buses are in use. Mathematically this becomes:

$$Tij = 1/3 \text{ Hr.}$$

$$Xij = 6 \text{ Bus/Hr.}$$

Then: $Tij \cdot Fij = (1/3 \text{ Hr.}) \cdot (6 \text{ Bus/Hr.}) = 2 \text{ Buses}$

Combining for all routes n during a time period j, the number of buses in use becomes:

$$\sum_{i=1}^{n} (Tij) \cdot (Xij)$$

Now considering that only Bj buses are available during time period j, it is clear that the above summation can never exceed Bj. or:

$$\sum_{i=1}^{n} (Tij) \cdot (Xij) \leq Bj \qquad \forall j$$

3.3.3 Cost Considerations

The cost considerations of todays bus systems are diverse and complex. The analysis presented here will condense and combine many factors to develop a usable and reliable yet general cost model. Precision is sacrificed for the speed and convenience of a simpler model, since the actual cost breakdown of this type of a system is a major task in itself.

Clearly, a relationship exists between the total daily cost of

operating a bus system and the number of hours operated daily. Individually, for each bus, as you increase the running time nearly all cost factors go up. The added costs accrue from increased driver wages, fuel and lubricant costs, maintenance costs and the cost of possible repairs. Further, the wear and tear on equipment, tires and supporting equipment also add to the costs. Finally, the fixed plant must be capable of handling the additional bus usage, if not then even more costs are accumulated. For more detail of a similar bus system's costs, see the work completed by Gary Urbanek. (41) That thesis shows the complexity and diversity of the cost considerations for a dial-a-ride system.

To describe the cost function of a conventional bus system in detail would be interesting but computationally burdensome. The function, including all of the above factors plus those not mentioned, would provide great accuracy but is unnecessary for the scope of this analysis. Fortunately, most bus companies have gone into their cost factors in sufficient depth to have a deeper understanding and have come up with a catch-all factor called cost per bus hour operated. This parameter combines all relevant costs and distributes them by operating time since this is the only system factor that may vary and therefore change the costs. Also, since in the review of bus costing it is noted that most costs are proportional to the number of hours of operation, it provides a very good measure, and represents

accurately, in one factor, all the diverse factors involved in cost estimation.

The daily cost of operating a bus system is simply the hourly cost (described above) times the total daily bus hours of operation. To determine daily bus hours operated, one must simply start at each route and compute route operating hours, combine and determine period operating hours and finally daily operating hours. In section 3.3.2 the total number of buses operating along all routes during a time period was established as;

$$\sum_{i=1}^{n} (Tij) \cdot (Xij)$$

Inserting the length of the time period in hours, Pj, the total number of bus hours operated during the period (j) may be found to be:

$$\sum_{i=1}^{n} (Tij) \cdot (Xij) \cdot (Pj)$$

Summing over all time periods gives the total daily bus-hours operated. It is:

$$\sum_{j=1}^{m} \sum_{i=1}^{n} (Tij) \cdot (Xij) \cdot (Pj)$$

Now, assuming knowledge of the cost per bus-hour factor, COST, the total daily cost may be found:

$$\text{Total Daily Cost} = \text{COST} \sum_{j=1}^{m} \sum_{i=1}^{n} (Tij) \cdot (Xij) \cdot (Pj)$$

3.3.4 Gross Income Determination

The total or gross income that a bus system earns comes primarily from the fare that the riders pay. A certain portion does come from subsidy but it is a constant amount and not a controllable variable therefore will be omitted. If it was included it would introduce a bias into the model and produce no useful results. For these reasons it is being neglected and left out of all further discussion here. This now reduces gross income to simply the revenue from rider fares.

The fare itself may vary from route to route in certain systems, due to longer distances or transfers, however, since the analysis only concerns itself with an urban bus system, and inter-route interactions have previously been neglected (chapter 2), a fixed fare is to be assumed. Any transfers are handled as separate riders, each paying full fare. The assumption is valid also because most urban systems do have a constant fare for nearly all rides. Model accuracy is not greatly hindered and ease of formulation is increased by this assumption.

The first step in finding income is to determine the total daily ridership. Since all demand is met (section 3.3.1) the ridership per hour along route i during time period j is simple R_{ij}, or if period j is P_j hours long, the total ridership along route i during period j is $R_{ij} \cdot P_j$. Summing over all routes, the total ridership during period j may be found. It is:

$$\sum_{i-1}^{n} (Rij)\cdot(Pj)$$

Summing again, this time over the time periods, gives the total daily riders. Therefore, total daily ridership is:

$$\sum_{j=1}^{m} \sum_{i=1}^{n} (Rij)\cdot(P\,j)$$

Assuming now that the fare is fixed at FARE dollars per rider, the daily gross income becomes

$$\text{Daily Gross Income} = \text{FARE} \sum_{j=1}^{m} \sum_{i=1}^{n} (Rij)\cdot(Pj)$$

3.3.5 Objective Function: Net Profit

The separate consideration of cost or gross income does not allow an accurate analysis since questions arise as to whether points of maximum income are actually points of maximum profit. For this reason profit is defined as the income less costs. In this analysis, since income and cost formulations are on a daily basis, it becomes:

Daily Profit = Daily Gross Income - Total Daily Cost.

From sections 3.3.3 and 3.3.4, mathematically this is:

$$\text{Prof} = \text{FARE} \sum_{j=i}^{m} \sum_{i=1}^{n} (Rij)\cdot(Pj) - \text{COST} \sum_{j=1}^{m} \sum_{i=1}^{n} (Tij)\cdot(Xij)\cdot(Pj)$$

or more simply:

$$PROF = \sum_{j=1}^{m} Pj \left[FARE \sum_{i=1}^{n} Rij - COST \sum_{i=1}^{n} (Tij)\cdot(Xij) \right]$$

This equation is to be considered the objective function of the formulation for the linear programming analysis.

3.3.6 Bounds on the Model

As a final portion of the system formulation a set of bounds must be specified in order to insure that the model, when solved by linear programming methods, actually represents the existing system. The solution is not, in itself, an attempt to determine the best state, it is merely a step along the way. By forcing the solution to represent the existing state, the output provides the required sensitivity analysis.

The bound set is simply the lower bounds on the frequencies along all routes. The bounds are specifically the existing frequencies along each route Xij^{o}

So: $Xij \geq Xij^{o} \qquad \forall ij$

By the use of this bound set, in the optimal solution to the above formulation the solution frequencies will at least meet the existing frequencies, and the solution itself will give an accurate representation of the system today. Through an interpretation of the sensitivity analysis that accompanies the solution, areas needing

change become obvious. If certain frequencies in the solution are above those existing presently, they are immediately applicable and will increase both profit and level of service.

3.4 Linear Approximation of Demand

As a final section of this chapter the linearization of the demand function will be shown. It must be included, since without it the entire formulation cannot be solved using linear programming techniques.

The concept of approximating a curve with linear segments is simple. Its use, however, when associated with linear programming is limited only by one prerequisite. That is that when maximizing a function, in this case profit, the constraint under consideration must describe a convex set in the relevant space. A convex set is defined as a set of points for which any line segment joining any two points of the set lies entirely within the set. The demand function, being an exponential, does meet this requirement since it describes a convex set.

The next step is to approximate the function with linear segments. Clearly the more segments used, the less error will develop, however, for each such segment used, the number of variables in the model is doubled. For this reason, and the basic inaccuracies inherent in this demand model, two segments are used. The first is sloped and the second horizontal, as shown below:

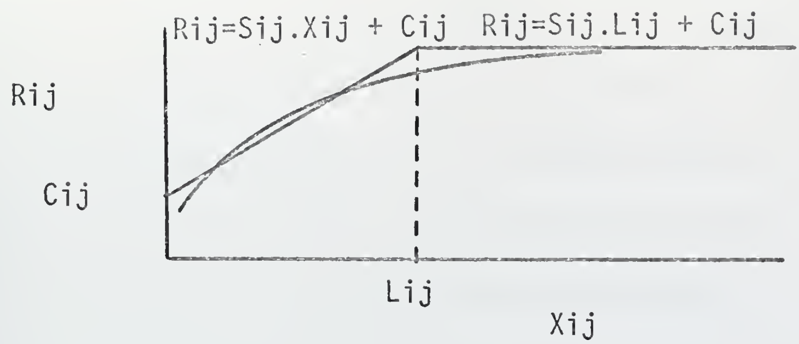

FIGURE 3.2 Linear Approximation of Demand Function

The sloped portion approximates the changes in ridership associated with changes in frequency. The slope Sij is the crucial factor here, it must provide the least amount of error possible when compared to the original demand function. The second portion describes the leveling off phenomenon. It begins at the frequency Lij, which is an approximation of the frequency beyond which changes to frequencies have no affect on ridership. By using these two linear segments the demand function has been approximated and the linear function still maintains the main characteristics of the original function, those being increasing ridership with increasing frequencies and the leveling off of ridership beyond certain frequencies.

The introduction of this two part demand function now requires the alteration of some of the variables. The original frequency Xij is now made up of two parts:

$$Xij = Xij1 + Xij2$$

The use of these two new variables may be eliminated if careful consideration of the system is made. In particular, beyond the point of increasing ridership, Lij, no additional bus service will be provided, since it only adds to cost and not to income, therefore, frequencies beyond the Lij values may be neglected.

Let $\quad Rij = Sij\ Xij \div Cij$

and $\quad Xij \leq Lij$

where $\quad Sij$ = empirical constant, slope of first linear segment

Lij = empirical constant, limit of first linear segment

Cij = empirical constant, positions first segment

3.5 Formulation Summary

Objective Function:

$$\text{Maximize PROF} = \sum_{j=1}^{m} Pj \left[\text{FARE} \sum_{i=1}^{n} Rij - \text{COST} \sum_{i=1}^{n} Tij \cdot Xij \right]$$

Subject:

$CAP \cdot Xij \geq Rij$	$\forall ij$	Capacity of Constraint
$\sum_{i=1}^{n} (Tij) \cdot (Xij) \leq Bj$	$\forall j$	Bus Availability Constraint
$Rij = Sij \cdot Xij + Cij$	$\forall ij$	Ridership Relationship
$Xij \geq Xij^{o}$	$\forall ij$	Bound Set

$$X_{ij} \geq 0 \qquad \forall ij$$

$$T_{ij} \geq 0 \qquad \forall ij$$

$$X_{ij} \leq L_{ij} \qquad \forall ij$$

Where:

P_j = Length of Period j (Hr)

CAP = Capacity of a Bus (Pax/Bus)

X_{ij} = Frequency Along Route i During Period J (Bus/Hr)

R_{ij} = Ridership Along Route i During Period J (Pax/Hr)

T_{ij} = Round Trip Travel Time for Route i During Period j (Hr)

B_j = Buses Available During Period j (Bus)

FARE = Fare of A Bus Ride ($)

COST = Cost per Bus Hour Operation ($/Bus-Hr)

S_{ij} = Scope of Linear Relationship Between Frequency and Ridership for Route i Time Period j (Pax-Hr/Bus)

L_{ij} = Empirical Constant for Route i During Period j (Bus/Hr)

C_{ij} = Empirical Constant for Route i During Period j (Pax/Hr)

$\overset{o}{X}_{ij}$ Existing Frequency Along Route i During Period j (Bus/Hr)

CHAPTER IV

INTERPRETATION OF RESULTS

4.1 Introduction

This chapter describes the interpretation of results from the linear programming solution of the formulation in chapter III. The model was a representation of the minimal existing system with the linear programming optimal solution simply another such representation. Associated with the computer solution of such a linear programming problem are many output reports. These reports include much useful information, such as a sensitivity analysis. The use of only two of the available reports is considered here since they can be used to develop short run changes.

In order to more clearly illustrate the concepts presented in the first portions of this chapter, a simple example is presented. The example is an analysis of a hypothetical three route bus system. The system contains all the facets of larger systems but is scaled down for simplicity and ease of illustration. Also, due to the reduced size of this problem, the use of the IBM 1130 computer is possible. Since this facility was readily available to the author, the discussion contained herein will deal with 1130 reports rather than the similar reports from the larger IBM 360 computer.

The interpretation of the two reports is the foundation upon which this entire analysis is built. The purpose of developing a linear

programming formulation that results in a representation of the present system as an optimal solution is solely to produce these reports. The reasoning behind this method of analysis will become clear at the completion of this chapter.

4.2 Discussion of Linear Programming Results

The goal of linear programming is to provide systematic methods by which one can fully analyze and comprehend the complex inter-relationships in a linear system. To more completely achieve this goal, aside from an optimal solution, the range of validity of the answer must be known. A modeller must know how far input parameter values may vary without causing violent changes in the computed solution. This type of analysis, termed sensitivity or post-optimality analysis, is the main feature discussed here and is the basis of this research.

The sensitivity analysis consists primarily of the dual variables and shadow prices. The dual variables are the variables in the dual, which is a problem associated with the primal (original) linear programming problem. The dual is simply a second or mirror image of the original, it appears arbitrary but provides the optimal values of the dual variables, which are the shadow prices. These values represent the unit worth of each resource as predicted in an optimal solution to the primal problem. They are

the associated values in the sensitivity analysis. Their existence is assured by the dual theorem, (16) which states that if a primal has an optimal feasible solution, so also does the dual.

The shadow prices allow three types of information to be determined. They show how to:

1) Find the range of variation in each objective function coefficient and right hand side constant over which the current optimal solution remains optimal.
2) Evaluate the economic impact of specific changes in the model's data.
3) Revise a previously optimal solution after adding new variables.

Of primary interest are the first two areas, the second being the more important. In order to be able to complete any of the above analyses, the determination of shadow prices is crucial. With the solution of the dual insured by the dual theorem, the values of the dual variables at optimality must exist. The dual variables are associated with each constraint and bound in the primal, those shadow prices which correspond with bounds give the range of variation, as described in number 1 above. The second area, economic impacts, comes directly from the shadow prices

associated with the constraints.

Since the pre-coded computer programs provide for the report of these values, the only factor remaining is to interpret the computer's results. The ease of obtaining these reports and the information that they present are the major factors that ideally suit them for short run system analysis.

4.3 Interpretation of L.P. Results

The availability of the IBM 1130 computer during the preparation of this analysis determined that for illustration purposes, the pre-coded linear programming package, LP MOSS, which is 1130 oriented, would be used. This program, along with the 1130 computer, is much slower and more limited in size than the MPS system on the IBM 360 computer. The LP MOSS package on the 1130 can only handle problems in which the product of the number of routes and the number of time periods is less than 175. For example, a system of 36 routes and 5 time periods would exceed the capabilities of the computer. However, for small systems such as the example, it is more than sufficient.

1130 LP MOSS provides the optimal solution to linear programming problems, however, it also is able to print many diverse output reports. Two are of primary interest here, LP SOLUTION and LP ANALYSIS, each providing useful data for the analysis being discussed. LP SOLUTION provides the basic solution

data such as variable usage and cost in the optimal solution. This information is the first step in the analysis of any system. LP ANALYSIS, on the other hand, actually provides the sensitivity analysis, it prints shadow prices under headings that indicate their meanings. The only task left to the analyst is to interpret these results and make rational decisions.

One additional point must be brought out here before going on to the actual-meanings of the outputs. The LP MOSS coding has been geared to the product-mix type of problem. In particular, a cost-minimization problem is the ideal use for LP MOSS. This means that nearly all values, while being labelled cost reductions or increases, actually indicate profit increases or decreases respectively. The profit maximazation nature of this analysis may cause some ambiguities but through careful consideration of values and signs, as well as meanings, there should be no problem.

4.3.1 Output Report: LP SOLUTION

The first output, LP SOLUTION, as shown in Figure 4.1, provides the solution to the stated problem. As seen in the figure, the output report has column headings along the top and variable names along the left side.

The variable names are indicative of their meaning. The Xij variables represent the frequency along route i during time period j.

VARIABLE	TYPE	ENTRIES	SOLUTION ACTIVITY	UPPER BOUND	LOWER BOUND	CURRENT COST	REDUCED COST
X11	B*	4	9.999	***********	6.000	64.799	0.000
PROF	B*	0	1470.959	***********	0.000	-1.000	-1.000
R11	B*	0	599.999	***********	0.000	0.000	0.000
Z11	B*	0	0.000	***********	0.000	0.000	0.000
BUS1	UL	0	20.000	20.000	0.000	0.000	-64.799
X12	B*	4	12.999	***********	4.000	43.199	0.000
R12	B*	0	519.999	***********	0.000	0.000	0.000
Z12	B*	0	4.333	***********	0.000	0.000	0.000
BUS2	UL	0	20.000	20.000	0.000	0.000	-43.199
X13	LL	4	4.000	***********	4.000	-16.200	-16.200
R13	B*	0	40.000	***********	0.000	0.000	0.000
Z13	B*	0	3.333	***********	0.000	0.000	0.000
BUS3	B*	0	8.000	20.000	0.000	0.000	0.000
X21	LL	4	4.000	***********	4.000	-20.699	-85.499
R21	B*	0	140.000	***********	0.000	0.000	0.000
Z21	B*	0	1.666	***********	0.000	0.000	0.000
X22	LL	4	3.000	***********	3.000	-14.400	-57.599
R22	B*	0	60.000	***********	0.000	0.000	0.000
Z22	B*	0	1.999	***********	0.000	0.000	0.000
X23	LL	4	2.000	***********	2.000	-18.360	-18.360
R23	B*	0	36.000	***********	0.000	0.000	0.000

FIGURE 4.1 OUTPUT REPORT: L.P. SOLUTION

Z23	B*	0	1.400	***********	0.000	C.000	0.000
X31	LL	4	6.000	***********	6.000	59.039	-5.759
R31	B*	0	288.000	***********	0.000	C.000	0.000
Z31	B*	0	1.200	***********	0.000	C.000	0.000
X32	LL	4	4.000	***********	4.000	35.639	-7.560
R32	B*	0	120.000	***********	0.000	C.000	0.000
Z32	B*	0	2.000	***********	0.000	C.000	0.000
X33	LL	4	2.000	***********	2.000	-3.960	-3.960
R33	B*	0	20.000	***********	0.000	C.000	0.000
Z33	B*	0	1.666	***********	0.000	C.000	0.000

FIGURE 4.1 (CONTINUED) OUTPUT REPORT: L.P. SOLUTION

PROF is the objective function name, it stands for the function being maximized in dollars. Rij indicates ridership on route i during period j in passengers per hour. BUSj gives the number of buses available during any time period j (in buses) and finally the Zij terms in the output report are simply dummy variables which are needed to code the formulation for LP MOSS.

The leading heading, "VARIABLE," is self-explanatory, representing the variable and the associated data in each row. The heading "TYPE" indicates, for each variable, whether the value of the variable in the optimal solution lies at an upper limit (UL), a lower limit (LL) or at an intermediate level (B*). It is merely an indicator for quick reference, to determine which variables are at bounds. The "ENTRIES" heading is not relevant to this analysis and will be neglected. The column heading is not relevant to this analysis and will be neglected. The column heading "SOLUTION ACTIVITY" shows the value of the variable used in the solution, it also shows the value of the objective function at optimality. The following two headings are also uncomplicated. "UPPER BOUND" and "LOWER BOUND" represent, for each variable the upper or lower limits respectfully specified in the formulation. Since negative values are infeasible values for variables, if no lower bound above zero is specified, the computer sets the lower limit at zero. Where no upper bounds are specified in the formulation, the variable may increase without bound (indicated

by an asterisk). The column headed "CURRENT COST" indicates the profit per unit variable as defined by the problem. This column simply reproduces the coefficient of the variable in the objective function.

The final heading, "REDUCED COST," requires somewhat more discussion since its significance is less obvious. The numbers found in this column represent the change in the objective function, or profit, per unit variable if the bound is relaxed. It provides significant information only if the associated variable in the solution is constrained at a bound. This is easily seen by looking for a UL or LL in the "TYPE" column. In other words, this column tells the reader exactly how the objective function will change if the binding constraint is relaxed. Here again, due to the slanting of LP MOSS toward cost-minimization problems, a negative value indicates an increase in profit. This fact is intuitively clear when examined closely. A negative sign implies minus cost or profit, so if cost changes in the negative direction, profit will increase.

The example at the end of this chapter illustrates the use of this information for system improvements.

4.3.2 Output Report: LP ANALYSIS

The second output report to be used is LP ANALYSIS (Figure 4.2). It provides similar, but considerably more useful information than LP SOLUTION. Composed of two parts, this report goes beyond the former in addressing the interactions of all variables with the objective function. The two portions of the report are identical in form, but one represents only variables constrained at bounds and the other only those variables which are at intermediate levels.

Across the top of the LP ANALYSIS report appear a series of headings describing the tabular information contained in the report. Note that the columns have two headings, and each variable listed is followed by two rows of information. In each case, the top heading corresponds to the information in the first row associated with each variable, and the second heading identifies the information in the second row.

The first six headings "VARIABLE," "TYPE," "SOLUTION ACTIVITY," "CURRENT COST," "UPPER BOUND," and "LOWER BOUND" are merely repetitions of the information found in the first report. This is provided for ease of reference and association. The following headings "COST/UNIT INCREASE" and "COST/UNIT DECREASE" are most important.

VARIABLES AT UPPER BOUND OR LOWER BOUND

VARIABLE	TYPE	SOLUTION ACTIVITY / CURRENT COST	UPPER BOUND / LOWER BOUND	COST/UNIT INCREASE / COST/UNIT DECREASE	INCREASED ACTIVITY / DECREASED ACTIVITY	LOWEST COST / HIGHEST COST
BUS1		20.000	20.000	-64.799	**********	**********
	UL	0.000	0.000	64.799	16.000	-64.799
BUS2		20.000	20.000	-43.199	**********	**********
	UL	0.000	0.000	43.199	11.000	-43.199
X13		4.000	**********	16.200	15.999	0.000
	LL	-16.200	4.000	-16.200	0.000	**********
X21		4.000	**********	85.499	7.999	64.799
	LL	-20.599	4.000	-85.499	-0.000	**********
X22		3.000	**********	57.599	11.999	43.199
	LL	-14.400	3.000	-57.599	0.000	**********
X23		2.000	**********	18.360	13.999	0.000
	LL	-18.360	2.000	-18.360	0.000	**********
X31		6.000	**********	5.759	9.999	64.799
	LL	59.039	6.000	-5.759	0.000	**********
X32		4.000	**********	7.560	12.999	43.199
	LL	35.639	4.000	-7.560	0.000	**********
X33		2.000	**********	3.960	13.999	0.000
	LL	-3.960	2.000	-3.960	0.000	**********

FIGURE 4.2 OUTPUT REPORT: LP ANALYSIS

VARIABLE	TYPE	SOLUTION ACTIVITY / CURRENT COST	UPPER BOUND / LOWER BOUND	COST/UNIT INCREASE / COST/UNIT DECREASE	INCREASED ACTIVITY / DECREASED ACTIVITY	LOWEST COST / HIGHEST COST
X11		9.999	***********	***********	9.999	***********
	B*	64.799	6.000	5.759	0.000	59.039
PROF		1470.959	***********	***********	1470.959	***********
	B*	-1.000	0.000	***********	1470.959	***********
R11		599.999	***********	***********	599.999	***********
	B*	0.000	0.000	0.095	360.000	-0.095
Z11		0.000	***********	***********	0.000	***********
	B*	0.000	0.000	***********	0.000	***********
X12		12.999	***********	***********	12.999	***********
	B*	43.199	4.000	7.560	-0.000	35.639
R12		519.999	***********	***********	519.999	***********
	B*	0.000	0.000	0.189	160.000	-0.189
Z12		4.333	***********	***********	4.333	***********
	B*	0.000	0.000	22.680	1.333	-22.680
R13		40.000	***********	1.620	159.999	1.620
	B*	0.000	0.000	***********	40.000	***********
Z13		3.333	***********	19.440	13.333	19.440
	B*	0.000	0.000	***********	3.333	***********
BUS3		8.000	20.000	3.959	***********	3.959

FIGURE 4.2 (CONTINUED) OUTPUT REPORT: LP ANALYSIS

	B*	0.000	0.000	***********	8.000	***********
R21		140.000	***********	2.442	279.999	2.442
	B*	0.000	0.000	***********	140.000	***********
Z21		1.666	***********	205.203	3.333	205.203
	B*	0.000	0.000	***********	1.666	***********
R22		60.000	***********	2.879	239.999	2.879
	B*	0.000	0.000	***********	60.000	***********
Z22		1.999	***********	86.400	7.999	86.400
	B*	0.000	0.000	***********	1.999	***********
R23		36.000	***********	1.020	251.999	1.020
	B*	0.000	0.000	***********	36.000	***********
Z23		1.400	***********	26.228	9.799	26.228
	B*	0.000	0.000	***********	1.400	***********
R31		288.000	***********	0.119	479.999	0.119
	B*	0.000	0.000	***********	288.000	***********
Z31		1.200	***********	28.799	1.999	28.799
	B*	0.000	0.000	***********	1.200	***********
R32		120.000	***********	0.252	389.999	0.252
	B*	0.000	0.000	***********	120.000	***********
Z32		2.000	***********	15.120	6.499	15.120
	B*	0.000	0.000	***********	2.000	***********
R33		20.000	***********	0.395	139.999	0.395
	B*	0.000	0.000	***********	20.000	***********
Z33		1.666	***********	4.752	11.666	4.752
	B*	0.000	0.000	***********	1.666	***********

FIGURE 4.2 (CONTINUED) OUTPUT REPORT: LP ANALYSIS

This information quantifies the relationships of the variables to the objective function. If the value of any specific variable could be changed, this column identifies the actual change in profit (objective function) that would result from a unit change. The negative sign again represents a profit increase due to the coding's pre-supposition of a cost-minimization problem. This information, when combined with the values in the next column, produces the basis for this analysis.

The columns "INCREASED ACTIVITY" and "DECREASED ACTIVITY" are used in conjunction with information from the previous and following columns to represent the range of validity of the solution. It also represents the limits to changes that may be made under the optimal solution. This means that profit increases (decreases) due to the variation of the input variable, as shown in the COST/UNIT INCREASE (DECREASE) column, will occur until the variable exceeds the range spec ified in the INCREASED (DECREASED) ACTIVITY column. Once these limits are exceeded, the optimal solution changes radically and cannot be predicted. In short, these columns give the range over which the analysis is valid.

The final columns "LOWEST COST" and "HIGHEST COST" indicate the sensitivity of the profit equation to changes in the coefficients of the related variables. If the coefficient of a variable changed to

the value shown in this column, then the solution activity for that variable would change to be the corresponding value shown in the INCREASED and DECREASED ACTIVITY column. To more easily understand the meaning of this information, simply insert the word profit for cost.

The next section will describe a simple system and go through the entire analysis procedure in order to illustrate the concepts presented above. The usefullness of this method for short run changes will become apparent through the use of this example.

4.4 Illustrative Example

The sample problem is a simple bus system of three routes that are serviced 18 hours a day. The day has been divided into three six hour periods: The peak hours, 0600 to 1000 and 1600 to 1800; the daytime period, 1000 to 1600 and the night period, 1800 to 2400. No bus service is provided during the hours 2400 to 0600.

Each route i has a current frequency during each time period j. Table 4.1 gives the values of the input variable Xij^{o}.

TIME PERIOD

ROUTE	1	2	3
1	6	4	4
2	4	3	2
3	6	4	2

$\overset{o}{X}ij$ in BUS/HR

TABLE 4.1 CURRENT FREQUENCIES

Beyond this, other fixed factors are FARE, which is 33¢ or \$.33; bus capacity, CAP = 60 PAX/BUS; cost per bus hour operated, COST = \$12/BUS-HR; and finally the bus availability during any time period. The value Bj = 20 Buses is fixed for all three periods for ease of demonstration. It may be assumed that all scheduled maintenance is completed during the period when no service is provided (2400 to 0600) and that the operator holds enough buses in reserve to maintain the availability of 20 buses per period.

The run times, Tij, along each route have also been established for each time period since rush hour congestion can cause delays and therefore differing round trip travel times.

		TIME PERIOD			
		1	2	3	
	1	.75	.5	.5	Tij in HOURS
ROUTE	2	1.25	.75	.75	
	3	.5	.33	.33	

TABLE 4.2 RUN TIME

A final, yet very important step, is the demand formulation. Referring to chapter III, the linearized form is used, and therefore requires the establishment of each slope and two constant terms. Since the 1130 LP MOSS coding places some severe restrictions on the input form, the demand function for each route must begin at the origin, this fixes the Cij terms at zero and reduces the accuracy of the analysis. Beyond this, an assumption will be made concerning the limit of demand generation along any route. It is that increases in bus service along any route beyond 20 buses per hour will produce no increases in ridership. What this simply means is that for analysis purposes, all Lij's are fixed at 20. The assumption is valid in this hypothetical case, but for a real system both the Cij and Lij would have to be empirically found. The final step is the establishment of the slopes of the linear demand function. Since this is a sample

problem, they will merely be stated here (Table 4.3), noting that this is a major portion of the pre-analysis work.

		TIME PERIOD			
		1	2	3	
	1	60	40	10	S_{ij} in PAX-HR/BUS
ROUTE	2	35	20	18	
	3	48	30	10	

TABLE 4.3 RIDERSHIP SLOPE

Summarizing and using the model of chapter III, the linear programming problem is developed. It is:

OBJECTIVE FUNCTION:

$$\text{MAXIMIZE} \qquad PROF = \sum_{j=1}^{3} P_j \left[FARE \sum_{i=1}^{3} R_{ij} - COST \sum_{i=1}^{3} T_{ij} \cdot X_{ij} \right]$$

SUBJECT TO:

$$CAP \cdot X_{ij} \geq R_{ij} \qquad \forall ij$$

$$\sum_{i=1}^{3} T_{ij} \cdot X_{ij} \leq B_j \qquad \forall j$$

$$R_{ij} = S_{ij} \cdot X_{ij} \qquad \forall ij$$

$$X_{ij} \geq \overset{o}{X}_{ij} \qquad \forall ij$$

$$X_{ij} \geq o \qquad \forall ij$$

WHERE:

$Pj = 6$ HOURS $\forall j$

CAP = 60 PAX/BUS

FARE = .33 \$

COST = 12 \$/BUS-HOUR

$Bj = 20$ BUSES $\forall j$

$Lij = 20$ BUS/HOUR $\forall ij$

$Cij = 0$ PAX $\forall ij$

Xij Defined in TABLE 4.1

Tij Defined in TABLE 4.2

Sij Defined in TABLE 4.3

Before beginning the analysis of the sample problem, certain information about the procedure must be presented. The output reports only indicate unit changes, the actual magnitude of the change to be implemented is at the discretion of the decision maker who is reviewing the reports. The changes presented here to the sample problem are the author's own, and should not be interpreted as the only or best changes possible. The author has attempted to choose those areas requiring most change and those which present the most informative alterations. Beyond this, he has not made any implications as to the general nature of changes for other systems.

To proceed with the analysis, the first step in the analysis procedure is to compare the solution frequencies with those existing. By simply comparing SOLUTION ACTIVITY with LOWER BOUND for each frequency any differences immediately indicate an area in which profit increases may take place. If the frequencies differ, the immediate implication is that some of the equipment has been standing idle during a period when it should have been in use. In the example, looking at figure 4.1, it is noted that the frequency along route 1 (during period 1) should be increased to the level of ten buses per hour as apposed to the six buses per hour now being run. Using a similar procedure all frequencies are checked in this manner and provide the following comparison:

		CURRENT FREQUENCIES			SOLUTION FREQUENCIES		
		TIME PERIOD			TIME PERIOD		
		1	2	3	1	2	3
ROUTE	1	6	4	4	10	13	4
	2	4	3	2	4	3	2
	3	6	4	2	6	4	2

(IN BUSES/HOUR)

TABLE 4.4 COMPARISON OF CURRENT TO SOLUTION FREQUENCIES

The information available from the above step, as shown in table 4.4 should not be assumed an optimal solution. The following step provides further insight into possible frequency changes that are not apparent above. The objective is to determine which bounds cause the most severe constraint to the objective function. The purpose of this is to find the areas which are bottlenecks to increased profit. In terms of the problem at hand, the analyst simply reads down the REDUCED COST column picking out the largest negative values and determines whether the variables associated with these values are of short run nature. If a binding constraint may be removed or reduced in the short run, another lever on short run profit has been found.

Picking the largest such negative term in the REDUCED COST column, one sees that the current frequency X21 (route 2 during period 1) of 4 is causing a loss of $85.50 per bus per hour. In other words, this constraint of 4 buses per hour is the most detrimental to profit. By reducing and redistributing the now available equipment in a more optimal manner, daily profit will increase by $85.50 at most. The determination of the proper route to which the additional available bus should be allocated is a simple matter of merely choosing that route in the same time period with the highest CURRENT COST (coefficient

of the row variable in the profit equation). In the example, since an additional bus (or more) is made available by the reduction of X21, the comparison of values in the CURRENT COST column for time period 1 indicates the most profitable route is 1. Clearly, since the reduction of X21 makes available extra buses they should be used to increase X11. If all coefficients are negative, as shown in the example for time period 3, it is most advantageous to reduce routes and allow the equipment to stand idle than be used along any route and produce additional costs rather than profits.

Similarly, changes such as an additional bus being available during period 1 would increase profit by $64.80 if placed on route 1. Also, reducing the frequency along route 2 during time period 2 to 2 buses per hour and using the additional bus on route 1 would increase profit by $57.60. In all cases the extra available buses should be placed on the routes with the highest positive CURRENT COST values. Table 4.5 summarizes all such changes and indicates the possible profit increases.

VARIABLE	CHANGE	INCREASE IN PROFIT
BUS 1	Increase buses available by 1 bus and use it on route 1.	64.80
BUS 2	Increase buses available by 1 bus and use it on route 1.	43.20

VARIABLE	CHANGE	INCREASE IN PROFIT
X13	Reduce frequency by 1 bus per hour and leave this additional bus idle	16.20
X21	Reduce frequency by 1 bus/per hour and use additional bus on route 1.	85.50
X22	Reduce frequency by 1 bus per hour and use additional bus on route 1.	57.60
X23	Reduce frequency by 1 bus per hour and leave this additional bus idle.	18.36
X31	Reduce frequency by 1 bus per hour and use additional bus on route 1.	5.76
X32	Reduce frequency by 1 bus per hour use additional bus on route 1	7.56
X33	Reduce frequency by 1 bus per hour and leave this additional bus idle.	3.96

TABLE 4.5 SUMMARY OF BOUND CHANGES

Turning now to the LP ANALYSIS report, the first procedure is to check the cost/unit increase or decrease column for large negative values. This column gives information about the sensitivity of the objective function to changes in any of the variables. The result is

the change in profit with a unit change in any variable. Of course, all previous changes determined from the LP Solution report (Table 4.5) will also appear here as a double check, however, other possibilities may also become apparent here. This procedure actually gives the analyst the quantative amounts that are associated with changes in any of the variables. In this way, areas of change may be chosen by their effect on profit.

In the example problem, looking at figure 4.2, it is clear that profit increases may be realized only by changing the bounded variables. However, the remainder of the variables, although not producing profit increases, may in evaluated in terms of least reduction in profit. For example the reduction of frequency X11 by one BUS/HOUR will reduce profit 5.76 dollars, while a similar reduction to X12 will cause a 7.56 dollar loss of profit. If either of these frequencies must be reduced, from the profit standpoint, X11 is most acceptable. The evaluation of the other variables with positive values in this column may be similarly accomplished.

The final step in the analysis is not made to produce changes but rather to help understand the interrelationships between variable coefficients and the solution activities. By reviewing the HIGHEST or LOWEST COST column and comparing it to the CURRENT COST and the INCREASED or DECREASED ACTIVITY columns, changes in the coefficients may be analyzed. It provides information that is useful if factors such as

FARE or COST change due to outside influence, such as changes in union wages or bus maintenance costs. If the coefficient is changed and reaches the HIGHEST (LOWEST) COST value then the solution activity for that variable will change until it corresponds with the respective DECREASED (INCREASED) ACTIVITY level. If, for example, the maintenance costs were cut through new technology which in turn, reduced the COST constant, this would change the coefficients of all variables in the objective function. For illustration purposes, assuming in particular that this change in COST changed the coefficient of the X21 from the previous value of -20.7 to 64.8, then as shown in the INCREASED ACTIVITY column the solution activity for this variable would now be 8 BUSES/HOUR as opposed to the old value of 4 BUSES/HOUR. This step is simply to further the understanding of system inter-relationships not to develop system alterations until some exogenous variable or constant is changed.

Summarizing the above steps the method is simply presented as a four step procedure:

1) Compare solution activity frequencies to existing frequencies in order to discover idle equipment.
2) Determine the most binding constraint to profit by reviewing the REDUCED COST and CURRENT COST columns.

3) Determine variable relationship to objective function by reviewing the COST/UNIT INCREASE (DECREASE) column.

4) Gain insight into coefficient relationships with profit using the LOWEST (HIGHEST) COST column.

In conclusion, the author will present the altered system, but prior to this, it must be established that the most profitable changes were first made to the extent possible, then the lesser changes implemented if they produced a profit increase greater than $10.00. The choice of 10 dollars for this value is arbitrary, but the author feels it represents a cut off point that must be made. To change frequencies and therefore schedules and timetables for a less than 10 dollar profit increase per bus seemed excessive, so such changes were neglected.

It was assumed that the only changes possible were the alteration of frequencies and the additional availability of one bus during any single time period. This latter assumption has been made to point out the possibility of making an additional bus available during a time period through the shifting of maintenance schedules. This type of change makes possible relatively large profit increases and may be well worth the effort. Table 4.6 summarizes the final bus system and compares it to the original one. The additional bus was made available during time period 1 since it is most beneficial there.

ORIGINAL BUS SYSTEM			
TIME PERIOD			
	1	2	3
ROUTE 1	6	4	4
2	4	3	2
3	6	4	2

REVISED BUS SYSTEM			
TIME PERIOD			
	1	2	3
ROUTE 1	14	15	1
2	1	1	1
3	6	4	2

TABLE 4.6 CHANGES SUMMARY AND COMPARISON

The changes shown above are all readily apparent from the computer output and all changes are of short run nature. The example, although simple enough to be solved by eye, served the purpose of illustrating the method.

CHAPTER V

SUMMARY AND CONCLUSIONS

5.1 Introduction

This thesis presented a method of analyzing a bus system through the use of linear programming. The individual parts of the method have existed for many years, however, their combination in this manner is unique. Briefly, this chapter will summarize the basic research and criticize the method.

The use of linear programming to produce optimal solutions is an established method. Associated with this use, the post-optimality analysis has always been an important factor, defining system responses to changes in the inputs. With this in mind, the author has revised the procedure so that the desired result is now the post-optimality reports rather than the optimal solution. To obtain this, the linear programming problem is constrained in such a way as to force the optimal solution to approximate the existing system. In this way, the sensitivity analysis is directly applicable to the system as it is now operating and identifies areas requiring change.

5.2 Summary

The problem selected for analysis was how to determine the short run changes necessary to improve the profitability, from level of service and profit aspects, of an existing bus system. The

importance of this problem is evident when comparing the financial condition of bus transit to any other mode. Buses are always on the short end of the financial stick. They seldom operate at a profit and never satisfy everyone. In order to improve either level of service or profit, some changes must be made. They should be the most beneficial, which are determined depending upon what measures of effectiveness are being used.

Beyond this, changes may be either of long or short run nature depending on the actions necessary to implement the changes. For example, the complete restructuring of the route network is a long run change as opposed to a simple frequency change which is possible in the short run. This thesis has only concerned itself with short run changes, since they must first be implemented in order to "buy time" to allow the implementation of long run changes.

The financial problems of bus systems arise from the nature of bus service in general. As discussed in Chapter I the factors affecting this problem are numerous and diverse, and require close scrutiny in order to single out the most detrimental. By using the computational power of linear programming this analysis may be quickly accomplished and the results applied in the short run.

The initial concept was to simply find an L.P. optimal solution and implement it in the short run. This proved impossible because such an optimal solution involved short and long run changes. With sucn

a combination the use of only a few of the changes could cause undesirable impacts. There was no guarantee that by the implementation of a portion of this optimal solution, the situation would improve beyond that existing presently. For this reason, the author's attention was turned to the post-optimality reports, the sensitivity analysis. These reports describe the impacts that will occur with the change of an input variable, as well as the range of validity over which these impacts are valid.

This helped develop the idea of constraining the actual L.P. formulation to represent the minimal existing system in the hope of developing a useful sensitivity analysis. The results were beneficial and directly applicable to the problem at hand. The post-optimality reports provided not only the sensitivity analysis hoped for, but also very useful insight into the system interactions which helped in the determination of change required.

The interpretation of the output reports is the basis upon which this analysis of a bus system is built. The understanding of the values incorporated therein is of prime importance and therefore must be complete or the analysis will fail, leaving no beneficial results.

The output reports, which accompany the linear programming solution of the formulation, are the means through which system changes become apparent. In particular, the use of the L.P. SOLUTION and L.P. ANALYSIS reports bring out the relationships between variables,

such as frequency, and the objective function. The changes that arise from this analysis are not only clearly identified but also quantified. Values are presented that correspond to unit changes in the variables, these in turn are evaluated by the analyst to determine the magnitude of change to be implemented.

5.3 Conclusions

The method presented in this thesis appears very useful in fulfilling the purpose of determining short run changes. However, since it has never been used on a large system that incorporates the various complex facets of bus service, its worth as an analytic technique has yet to be proven. Hopefully, in the future, the opportunity will arise to use this method, and its results will be found useful.

More specifically, concerning the method, a great deal of demand data is required. The empirical data needed to determine the Cij, Lij and Sij terms is extensive and must be accurate for each route during each time period in order to provide the necessary modelling precision. The compilation of this data, in itself could invalidate this method as a short run technique. However, if the data is already available, such as through past studies, the method is immediately applicable.

The memory space of the various computers also can become a factor when working with larger systems. The IBM 1130 was suitable for the three route example problem of chapter IV, but would not have sufficient

memory for a two hundred route network. Past trends in computer technology indicate that the computer capacity will be sufficient for this technique to be applicable in the future.

Finally, a question arises as to the necessity of actually running the computer L.P. program once the formulation has been developed. Since the system model is linear, the coefficients will always indicate areas that require change or will provide most increases in profit when changed. Most likely, in very large systems, the post-optimality reports will be useful, but the information may also be available through numerical techniques, eliminating the requirement for the computer completely. This again, must be determined when the method is actually used on a large scale system.

The application of this technique to a large bus system as opposed to the small system presented here, will create problems of the nature discussed above. In particular, the demand modelling portion, due to the data requirements, may become excessive for a large system. The system modelling will pose no problems, however, since it is simply an extension of the model described in chapter III. Beyond this, the computer solution would be useful in the analysis of the large system because it presents all relevant interactions and impacts in a concise manner. The burden still lies with the analyst, however, to interpret the computer results and implement changes.

Extending this method to apply to other systems is a simple

procedure. Any system which can be modelled in linear form may be analyzed using the technique presented here. The procedure would be to model the system, again taking note to constrain the formulation at the minimal or existing situation, and proceed as discussed in chapter IV. The applicability of this technique to other systems provides interesting theoretical material, however, until its validation in the bus context, its results must be carefully reviewed by all potential users.

In summary, the method appears useful, but must be tested in order to prove its validity. The formulation itself has included many assumptions, such as good headway control and no route interaction, which are not actually correct, but have aided in modelling simplicity. The demand formulation is also weak due to the elimination of all relevant variables except frequency. The factors discussed in chapter II all play a part in forming demand, however, due to their being vague and unquantifiable associated with the short run time frame of this analysis, they have been eliminated.

These problems introduce error as well as modelling inaccuracies that may combine to limit the validity of the model. All of these considerations must be weighed and measured by any potential users in order to determine the actual use made of the technique. The author, however, feels that even considering the drawbacks of the method, it still provides a useful means of determining short run changes to bus systems.

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